CBSE CLASS X

Mathematics Standard (041)

ANSWER KEY

From previous CBSE Board Exam questions

Code: AA7248	**Questions: 15**	**Maximum Marks: 31**	**Generated: 2026-06-15 13:05**

SELECTIONS USED

Source	Previous-year board
Subject	Mathematics
Lessons	Proofs in Mathematics
Year	2022-2026
Questions selected	15

Composition — Types: 6 Short · 4 Very short

Q1. § 1.3 Revisiting Irrational Numbers **[3]**

Prove that $\sqrt{2}$ is an irrational number.

Previously asked in: 2025 30/4/1 Q26(A); 2023 30/6/1 Q27(B) — **2×**

◆ **Real Numbers** | 1.3 Revisiting Irrational Numbers | A1.7 Proof by Contradiction

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Model Answer

Proof: Assume, to the contrary, that $\sqrt{2}$ is rational.

Then we can find integers a and b ($b \neq 0$) such that $\sqrt{2} = \frac{a}{b}$, where a and b are coprime (no common factor other than 1).

Squaring both sides: $2b^2 = a^2$

So 2 divides a^2. By Theorem 1.2, 2 divides a.

Let $a = 2c$ for some integer c. Substituting:

$$2b^2 = 4c^2 \implies b^2 = 2c^2$$

So 2 divides b^2, which means 2 divides b.

Thus 2 is a common factor of both a and b, which **contradicts** the fact that a and b are coprime.

This contradiction arose because of our incorrect assumption. Hence, $\sqrt{2}$ is irrational.

Source: Chapter 1, Section 1.3 – Revisiting Irrational Numbers

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Explanation

- This is a **proof by contradiction** — always start by assuming the opposite of what you want to prove.
- Key step: use **Theorem 1.2** (if a prime p divides a^2, then p divides a) — examiners expect you to cite this.
- The contradiction must be clearly stated: a and b were assumed coprime, but both are divisible by 2.
- Write "Hence $\sqrt{2}$ is irrational" as the concluding line — never omit the conclusion.

Q2. § 1.3 Revisiting Irrational Numbers **[2]**

Prove that $2 - 5\sqrt{3}$ is an irrational number given that $\sqrt{3}$ is irrational.

Previously asked in: 2026 30/5/1 Q21

◆ **Real Numbers** | 1.3 Revisiting Irrational Numbers | A1.7 Proof by Contradiction

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Model Answer

Assume, to the contrary, that $2 - 5\sqrt{3}$ is rational.

Then there exist coprime integers a and b ($b \neq 0$) such that:

$$2 - 5\sqrt{3} = \frac{a}{b}$$

$$\Rightarrow 5\sqrt{3} = 2 - \frac{a}{b} = \frac{2b - a}{b}$$

$$\Rightarrow \sqrt{3} = \frac{2b - a}{5b}$$

Since a and b are integers, $\frac{2b - a}{5b}$ is rational, which means $\sqrt{3}$ is rational.

But this **contradicts** the given fact that $\sqrt{3}$ is irrational.

Therefore, our assumption is wrong and $2 - 5\sqrt{3}$ is **irrational**.

Source: Chapter 1, Section 1.3 — Revisiting Irrational Numbers

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Explanation

- This is a **proof by contradiction** question — always start by assuming the opposite of what you want to prove.
- The key step is isolating $\sqrt{3}$ on one side to show it would have to be rational — contradicting the given condition.
- Examiners award marks for: (1) correct assumption, (2) algebraic rearrangement to isolate $\sqrt{3}$, (3) stating the contradiction clearly. Don't skip the contradiction statement.

Q3. § 1.3 Revisiting Irrational Numbers § 1.4 Summary **[3]**

Prove that $\sqrt{3}$ is an irrational number.

Previously asked in: 2026 30/1/1 Q26; 2025 30/2/1 Q30; 2025 30/5/1 Q26(a); 2024 30/5/1 Q26(a) (OR-1); 2023 30/5/1 Q26 (OR-1) — **5**×

◆ **Real Numbers** 1.3 Revisiting Irrational Numbers A1.7 Proof by Contradiction

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Model Answer

Proof: Assume, to the contrary, that $\sqrt{3}$ is rational.

Then we can find integers a and b ($b \neq 0$) such that $\sqrt{3} = \frac{a}{b}$, where a and b are coprime (no common factor other than 1).

So, $b\sqrt{3} = a$.

Squaring both sides: $3b^2 = a^2$

Therefore, a^2 is divisible by 3, and by Theorem 1.2, a is also divisible by 3.

So, let $a = 3c$ for some integer c.

Substituting: $3b^2 = 9c^2 \Rightarrow b^2 = 3c^2$

This means b^2 is divisible by 3, so b is also divisible by 3.

Therefore, a and b have at least 3 as a common factor. But this **contradicts** the fact that a and b are coprime.

This contradiction arose because of our incorrect assumption. Hence, $\sqrt{3}$ is irrational. ■

Source: Chapter 1, Section 1.3 – Revisiting Irrational Numbers

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Explanation

- This is a **proof by contradiction** — always start by assuming the opposite of what you want to prove.
- The key tool used is **Theorem 1.2**: if a prime p divides a^2, then p divides a.
- The critical step students miss: after substituting $a = 3c$, show that **3 also divides** b — this creates the contradiction with "coprime."
- Write the conclusion clearly: state the contradiction and then state that $\sqrt{3}$ is irrational.
- Examiners award marks for: assumption (½), algebraic steps (1), applying Theorem 1.2 twice (½), contradiction + conclusion (1).

Q4. § 1.3 Revisiting Irrational Numbers § 1.4 Summary **[3]**

Prove that $4 + 3\sqrt{2}$ is an irrational number given that $\sqrt{2}$ is an irrational number.

Previously asked in: 2025 30/3/1 Q27

◆ **Real Numbers** 1.3 Revisiting Irrational Numbers A1.7 Proof by Contradiction

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Model Answer

Proof (by contradiction):

Assume, to the contrary, that $4 + 3\sqrt{2}$ is rational.

Then we can find coprime integers a and b ($b \neq 0$) such that:

$$4 + 3\sqrt{2} = \frac{a}{b}$$

Rearranging:

$$3\sqrt{2} = \frac{a}{b} - 4 = \frac{a - 4b}{b}$$

$$\sqrt{2} = \frac{a - 4b}{3b}$$

Since a and b are integers, $\frac{a - 4b}{3b}$ is rational, which means $\sqrt{2}$ is rational.

But this **contradicts** the given fact that $\sqrt{2}$ is irrational.

This contradiction arose because of our incorrect assumption.

$\therefore 4 + 3\sqrt{2}$ is irrational. ■

Source: Chapter 1, Section 1.3 – Revisiting Irrational Numbers

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Explanation

- Use **proof by contradiction**: assume the number is rational, express it as $\frac{a}{b}$ with a, b coprime.
- Isolate $\sqrt{2}$ algebraically — it ends up equal to a rational expression, contradicting the given condition.
- Always state clearly *what* the contradiction is and *why* it arose.
- Examiners award marks for: correct assumption (1), correct algebraic rearrangement to isolate $\sqrt{2}$ (1), stating the contradiction and conclusion (1).

Q5. § 1.3 Revisiting Irrational Numbers **[3]**

Prove that $\frac{2-\sqrt{3}}{5}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

Previously asked in: 2024 30/4/1 Q30

◆ **Real Numbers** | 1.3 Revisiting Irrational Numbers | A1.7 Proof by Contradiction

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Model Answer

Proof by contradiction.

Assume, to the contrary, that $\frac{2-\sqrt{3}}{5}$ is rational.

Then we can find coprime integers a and b ($b \neq 0$) such that:

$$\frac{2-\sqrt{3}}{5} = \frac{a}{b}$$

Rearranging:

$$2-\sqrt{3} = \frac{5a}{b}$$

$$\sqrt{3} = 2 - \frac{5a}{b} = \frac{2b-5a}{b}$$

Since a and b are integers, $\frac{2b-5a}{b}$ is rational, which means $\sqrt{3}$ is rational.

But this **contradicts** the given fact that $\sqrt{3}$ is irrational.

This contradiction arose because of our incorrect assumption.

Therefore, $\frac{2-\sqrt{3}}{5}$ is irrational. ■

Source: Chapter 1, Section 1.3 Revisiting Irrational Numbers

Explanation

- The key technique is **proof by contradiction**: assume the number is rational, then algebraically isolate $\sqrt{3}$ and show it would have to be rational — contradicting the given condition.
- Always state that a and b are integers to justify that the rearranged expression is rational.
- Examiners award marks for: (1) correct assumption, (2) correct algebraic rearrangement to isolate $\sqrt{3}$, (3) citing the contradiction with the given fact, and (4) the conclusion. Don't skip any step.

Q6. § 1.3 Revisiting Irrational Numbers **[2]**

Prove that $5 - 2\sqrt{3}$ is an irrational number. It is given that $\sqrt{3}$ is an irrational number.

Previously asked in: 2024 30/1/1 Q25(A)

◆ **Real Numbers** | 1.3 Revisiting Irrational Numbers | A1.7 Proof by Contradiction

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Model Answer

Proof (by contradiction):

Assume $5 - 2\sqrt{3}$ is rational. Then we can write:

$$5 - 2\sqrt{3} = \frac{a}{b}, \quad \text{where } a, b \text{ are integers, } b \neq 0$$

Rearranging:

$$2\sqrt{3} = 5 - \frac{a}{b} = \frac{5b - a}{b}$$

$$\sqrt{3} = \frac{5b - a}{2b}$$

Since a and b are integers, $\dfrac{5b - a}{2b}$ is rational, which means $\sqrt{3}$ is rational.

But this **contradicts** the given fact that $\sqrt{3}$ is irrational.

Therefore, our assumption is wrong, and $5 - 2\sqrt{3}$ is **irrational**. ■

Source: Real Numbers, Section 1.3 – Revisiting Irrational Numbers

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Explanation

- Use **proof by contradiction**: assume the number is rational, express it as $\frac{a}{b}$, and rearrange to isolate $\sqrt{3}$.
- The key step is showing this forces $\sqrt{3}$ to be rational — contradicting the given condition.
- Examiners award marks for: (1) correct assumption, (2) correct algebraic rearrangement to isolate $\sqrt{3}$, (3) stating the contradiction clearly.
- Don't forget to state the conclusion explicitly.

Q7. § 1.3 Revisiting Irrational Numbers § 1.4 Summary **[3]**

Prove that $\left(\sqrt{2}+\frac{\sqrt{3}}{2}\right)$ is an irrational number, given that $\sqrt{6}$ is an irrational number.

Previously asked in: 2024 30/5/1 Q26(b) (OR-2)

◆ **Real Numbers** 1.3 Revisiting Irrational Numbers A1.7 Proof by Contradiction

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Model Answer

Assume, to the contrary, that $\left(\sqrt{2}+\frac{\sqrt{3}}{2}\right)$ is rational.

Then there exist coprime integers a and b ($b \neq 0$) such that:

$$\sqrt{2}+\frac{\sqrt{3}}{2}=\frac{a}{b}$$

Rearranging:

$$\frac{\sqrt{3}}{2}=\frac{a}{b}-\sqrt{2}$$

$$\sqrt{3}=2\left(\frac{a}{b}-\sqrt{2}\right)=\frac{2a}{b}-2\sqrt{2}$$

$$2\sqrt{2}=\frac{2a}{b}-\sqrt{3}$$

$$\sqrt{6}=\frac{1}{2}\left(\frac{2a}{b}-\sqrt{3}\right)\cdot\sqrt{2}$$

A simpler path: squaring both sides of $\sqrt{2}+\frac{\sqrt{3}}{2}=\frac{a}{b}$:

$$2+\sqrt{6}+\frac{3}{4}=\frac{a^2}{b^2}$$

$$\sqrt{6}=\frac{a^2}{b^2}-\frac{11}{4}=\frac{4a^2-11b^2}{4b^2}$$

Since a, b are integers, the RHS $\frac{4a^2-11b^2}{4b^2}$ is **rational**.

This means $\sqrt{6}$ is rational — **a contradiction**, since $\sqrt{6}$ is given to be irrational.

Hence, our assumption is wrong, and $\left(\sqrt{2}+\frac{\sqrt{3}}{2}\right)$ is **irrational**. ■

Source: Chapter 1, Section 1.3 – Revisiting Irrational Numbers

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Explanation

- The standard method is **proof by contradiction**: assume the expression is rational, then algebraically isolate a known irrational ($\sqrt{6}$ here) and show it would have to be rational — contradiction.

- The key step is squaring: $\left(\sqrt{2}+\frac{\sqrt{3}}{2}\right)^2 = 2+\sqrt{6}+\frac{3}{4} = \frac{11}{4}+\sqrt{6}$, which lets you isolate $\sqrt{6}$.
- Always explicitly state "this contradicts the fact that $\sqrt{6}$ is irrational" — examiners award a mark specifically for this conclusion.
- The given condition "$\sqrt{6}$ is irrational" **must** be used; never leave it unused.

Q8. § 1.3 Revisiting Irrational Numbers § 1.4 Summary **[2]**

Prove that $2+\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

Previously asked in: 2023 30/2/1 Q21

◆ **Real Numbers** 1.3 Revisiting Irrational Numbers A1.7 Proof by Contradiction

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Model Answer

Proof (by contradiction):

Assume $2+\sqrt{3}$ is rational. Then we can write:

$$2+\sqrt{3} = \frac{a}{b}, \quad \text{where } a, b \text{ are integers and } b \neq 0$$

Rearranging:

$$\sqrt{3} = \frac{a}{b} - 2 = \frac{a-2b}{b}$$

Since a and b are integers, $\frac{a-2b}{b}$ is rational, which means $\sqrt{3}$ is rational.

But this **contradicts** the given fact that $\sqrt{3}$ is irrational.

Therefore, our assumption is wrong, and $2+\sqrt{3}$ is **irrational**. ■

Source: Chapter 1, Section 1.3 – Revisiting Irrational Numbers

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Explanation

- This is a standard "proof by contradiction" question. Always start by **assuming the opposite** of what you want to prove.
- The key step is **isolating** $\sqrt{3}$ on one side — showing it equals a rational expression, which contradicts the given condition.
- Examiners award marks for: (1) correct assumption, (2) correct rearrangement isolating $\sqrt{3}$, (3) stating the contradiction clearly, (4) conclusion. Don't skip the conclusion line.

Q9. § 1.3 Revisiting Irrational Numbers **[3]**

Prove that $\sqrt{5}$ is an irrational number.

Previously asked in: 2026 30/2/1 Q26; 2026 30/3/1 Q28; 2025 30/1/1 Q29; 2025 30/6/1 Q26(a); 2024 30/2/1 Q27; 2023 30/4/1 Q27; 2023 30/1/1 Q27 — 7×

◆ **Real Numbers** | 1.3 Revisiting Irrational Numbers | A1.7 Proof by Contradiction

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Model Answer

Proof: Assume, to the contrary, that $\sqrt{5}$ is rational.

Then we can find integers a and b ($b \neq 0$) such that $\sqrt{5} = \frac{a}{b}$, where a and b are coprime.

Squaring both sides: $5b^2 = a^2$

So 5 divides a^2. By Theorem 1.2, 5 divides a.

Write $a = 5c$ for some integer c. Substituting:

$5b^2 = 25c^2 \Rightarrow b^2 = 5c^2$

So 5 divides b^2, and therefore 5 divides b.

Thus 5 is a common factor of a and b, **contradicting** the fact that a and b are coprime.

This contradiction arose because we assumed $\sqrt{5}$ is rational. Hence, $\sqrt{5}$ is irrational. ■

Source: Chapter 1, Section 1.3 – Revisiting Irrational Numbers

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Explanation

- This is a standard **proof by contradiction**. Always start by assuming the opposite of what you want to prove.
- The key tool is **Theorem 1.2**: if a prime p divides a^2, then p divides a. Examiners expect you to cite or use this explicitly.
- The contradiction must target the **coprime condition** — that is where the marks are.
- Write the substitution step ($a = 5c$) clearly; missing it loses a mark.

Q10. [2]

⚠ Proofs in Mathematics — Enrichment material — not examinable in the Board exam (per CBSE 2026-27 curriculum) NCERT enrichment appendix; not part of the examinable curriculum (omitted from the Course Structure). [source: Course Structure (appendices not listed)]

Show that the number $5 \times 11 \times 17 + 3 \times 11$ is a composite number.

Previously asked in: 2024 30/1/1 Q25(B)

◆ **Proofs in Mathematics** Composite Numbers and Factorization Proofs

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Model Answer

$5 \times 11 \times 17 + 3 \times 11$

$= 11(5 \times 17 + 3)$

$= 11(85 + 3)$

$= 11 \times 88$

$= 11 \times 8 \times 11$

$= 8 \times 11^2$

Since the number can be expressed as a product of factors (other than 1 and itself), it is a **composite number**.

Source: Chapter 1, The Fundamental Theorem of Arithmetic

Explanation

- The key step is **taking 11 as a common factor**. This immediately shows the number is divisible by 11 (and also by 8), so it has factors other than 1 and itself — the definition of a composite number.
- Examiners expect you to show the factorisation step-by-step and explicitly state the conclusion. Do not just calculate the final value (968); show *why* it is composite by exhibiting its factors.
- This is similar to Exercise 1.1, Q.6 from Chapter 1.